OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Advanced Subsidiary General Certificate of Education
Advanced General Certificate of Education

MEI STRUCTURED MATHEMATICS 2609

Mechanics 3

Wednesday **22 JUNE 2005** Afternoon 1 hour 20 minutes

Additional materials:
Answer booklet
Graph paper
MEI Examination Formulae and Tables (MF12)

TIME 1 hour 20 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Answer **all** questions.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The allocation of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- Final answers should be given to a degree of accuracy appropriate to the context.
- Take $g = 9.8 \text{ m s}^{-2}$ unless otherwise instructed.
- The total number of marks for this paper is 60.

This question paper consists of 5 printed pages and 3 blank pages.

1 A light elastic band has a natural length of $6a$ (i.e. it will just fit round the perimeter of a square of side $1.5a$ without being stretched).

Initially the band is stretched round four small, smooth pegs at the vertices of a square in a vertical plane. The square has side $2a$. The stiffness of the elastic is $\frac{mg}{a}$. This situation is shown in Fig. 1.1.

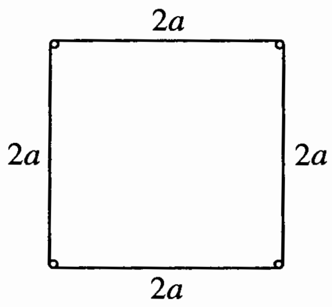


Fig. 1.1

(i) Find the tension in the band. Find also the energy stored in it. [3]

A particle of mass m is attached to the band at the mid-point of the lowest side of the square. The particle is lowered a distance h until the system is at rest in equilibrium, as shown in Fig. 1.2.

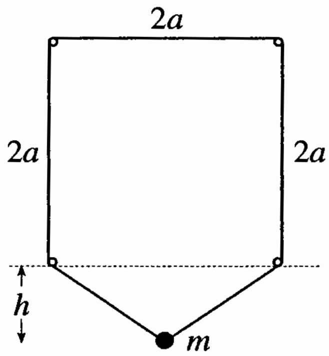


Fig. 1.2

(ii) Show that the extension in the band is $2\sqrt{a^2 + h^2}$ and find the value of h. [6]

The particle is pulled vertically downwards a further distance h and released from rest.

(iii) Find the speed of the particle when it passes through its equilibrium position. [7]

[Total 16]

2 **(a)** The speed, v, of a particle whose motion is simple harmonic is given by

$$v^2 = \omega^2(a^2 - x^2),$$

where x is the displacement from the centre of the motion, a is the amplitude and ω is a further constant.

(i) State the dimensions of a, x and v and hence establish the dimensions of ω. [3]

(ii) Name a quantity with the same dimensions as ω. [1]

(b) A fisherman's float of length 0.15 m has a mass of 0.015 kg and moves in a vertical line. It is at rest in equilibrium when the bottom of the float is 0.098 m under water.

In a general position, the bottom of the float is y m under water and is x m below the equilibrium position, as shown in Fig. 2. The upward force on the float, F N, is directly proportional to y. You may assume that the float is never completely immersed.

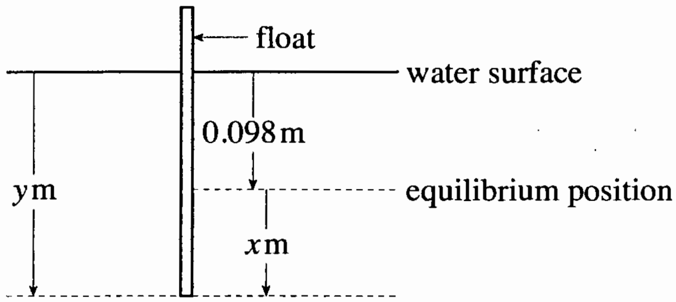


Fig. 2

(i) Show that $F = 1.5y$. [2]

The float is pulled down below the equilibrium position and released from rest.

(ii) By using Newton's second law, show that the equation of motion is $\ddot{x} + 100x = 0$. [4]

The float is released from rest 0.02 m below its equilibrium position.

(iii) Calculate the speed of the float when it is 0.01 m above its equilibrium position and moving **downwards**. Calculate also the time taken to reach this position. [5]

[Total 15]

3 A smooth, horizontal circular disc of radius 0.4 m has a rough vertical rim. Initially, the disc is rotating about its centre with a constant angular speed of $10\,\mathrm{rad\,s^{-1}}$ with a particle of mass 0.3 kg touching the rim, as shown in Fig. 3. The coefficient of friction between the particle and the rim is μ and the particle does not slip.

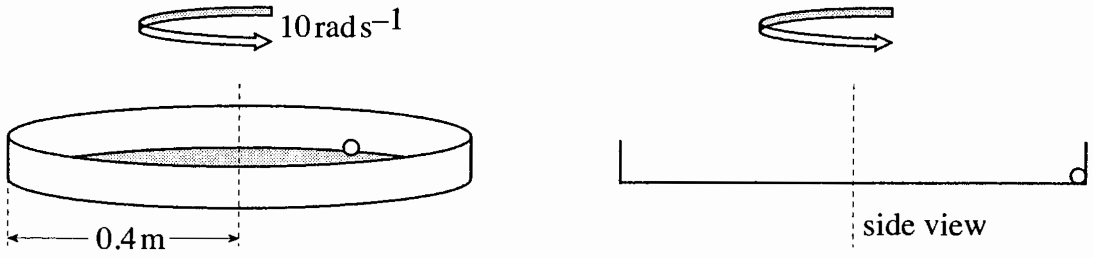


Fig. 3

(i) Explain briefly why there is no horizontal frictional force between the particle and the rim. [1]

(ii) Calculate the normal reaction of the rim on the particle. [2]

(iii) Find the least possible value of μ so that the particle could travel in a horizontal circle in contact with the rim but not in contact with the horizontal disc. [3]

The disc is now accelerated. All points on the rim have a tangential acceleration of $2\,\mathrm{m\,s^{-2}}$ so that the angular speed, t seconds after the acceleration starts, is $(10 + 5t)\,\mathrm{rad\,s^{-1}}$. The particle is in contact with the smooth horizontal disc and with the rim. The vertical frictional force is zero.

(iv) Show that the normal reaction, R N, of the rim on the particle at time t is given by

$$R = 3(2 + t)^2.$$ [2]

(v) Show that, in order to prevent slipping at time t,

$$\mu \geqslant \frac{1}{5(2 + t)^2}.$$

Deduce that the particle will not slip if $\mu \geqslant 0.05$, explaining your reasoning clearly but briefly. [5]

[Total 13]

4 The region R of a plane is bounded by the circle with equation $x^2 + y^2 = r^2$, the x-axis, the y-axis and the line $x = h, 0 < h \leqslant r$, as shown in Fig. 4.1. A uniform solid S is in the shape of R rotated through 4 right angles about the x-axis.

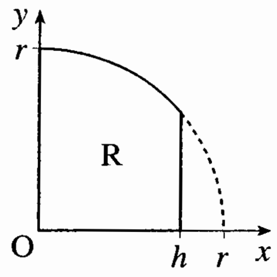


Fig. 4.1

(i) Calculate the volume of S and show that the distance of its centre of mass from the centre of the larger plane face is $\dfrac{3h(2r^2 - h^2)}{4(3r^2 - h^2)}$. [8]

(ii) Show that your result in part **(i)** agrees with the standard result for the position of the centre of mass of a uniform solid hemisphere. [1]

(iii) Show that, when $h = \frac{1}{2}r$, the solid S has volume $\dfrac{11\pi r^3}{24}$ and its centre of mass is $\dfrac{21r}{88}$ from the larger plane face. [2]

A solid, uniform hemisphere of radius r is divided into parts A and B by a plane parallel to the plane face cutting the radius perpendicular to this face in half, as shown in Fig. 4.2.

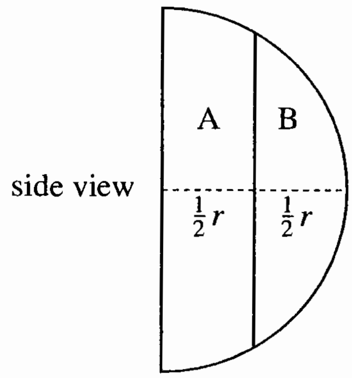
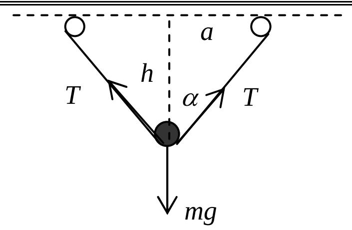


Fig. 4.2

(iv) Find the distance of the centre of mass of the solid B from the centre of its plane face. [5]

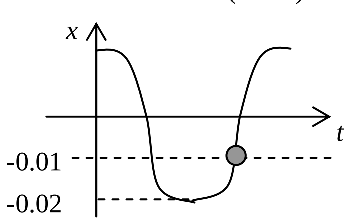
[Total 16]

Mark Scheme 2609
June 2005

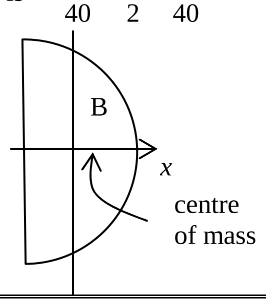
Q 1		mark		sub
(i)	$T = \frac{mg}{a}(8a - 6a) = 2mg$	B1	Use of HL	
	$E = \frac{mg}{2a}(2a)^2$	M1	Use of $0.5k\varepsilon^2$	
	$= 2mga$	F1	FT extension used in calculation of T	3
(ii)				
	Extension is $2\sqrt{a^2 + h^2} + 6a - 6a = 2\sqrt{a^2 + h^2}$	E1	Need some reference to +6a – 6a (accept words).	
	$\uparrow \quad 2T\cos\alpha = mg \quad (1)$	B1	Resolve vertically	
	$T = \frac{mg}{a} \times 2\sqrt{a^2 + h^2}$	B1	Or equivalent	
	$\cos\alpha = \frac{h}{\sqrt{a^2 + h^2}}$	B1		
	Substituting in (1) $2 \times \frac{2mg}{a}\sqrt{a^2 + h^2} \times \frac{h}{\sqrt{a^2 + h^2}} = mg$	M1	Eliminating **their** T .	
	so $h = \frac{a}{4}$	A1		6
(iii)	**either**			
	Put $h = \frac{a}{4}$ gives extensions		h need not be substituted	
	at equilib pos $2\sqrt{\frac{17a^2}{16}}$, at bottom $2\sqrt{\frac{5a^2}{4}}$	M1	Attempt to find extension and energy in terms of a and/or h	
	EPE at equilib pos $\frac{17mga}{8}$, $\left[\frac{2mg}{a}\left(a^2 + h^2\right)\right]$	F1	Equilib EPE. If h substituted FT from (ii)	
	EPE at bottom $\frac{5mga}{2}$, $\left[\frac{2mg}{a}\left(a^2 + 4h^2\right)\right]$	F1	Bottom EPE. If h substituted FT from (ii)	
	$\frac{5mga}{2} = \frac{mga}{4} + \frac{17mga}{8} + \frac{1}{2}mv^2$	M1	W-E. Accept EPE at equilib and GPE missing. Accept EPE using Δ ext	
		A1	All present. Accept sign errors and FT **their** values	
		B1	GPE term. Accept sign error. If h subst FT from (ii)	
	so $v^2 = \frac{1}{4}ag$ and $v = \frac{1}{2}\sqrt{ag}$ $\left(\approx 1.57\sqrt{a}\right)$	A1	cao. Accept either form.	

	$\left[\sqrt{\frac{2hg}{a}(6h-a)}\right]$			7

Q 1	continued	mark		sub
	or			
	Consider the SHM		h need not be substituted	
	ext from equilib $y \downarrow$			
	$mg - 2T\cos\alpha = m\ddot{y}$	M1	Attempt at an equation of motion in the vertical direction. Must have weight. T need not be resolved.	
		F1	T correct and resolved	
		A1	All correct	
	giving $\ddot{y} + \frac{4g}{a}y = 0$	A1	cao . Accept any form.	
	$v = a\omega$ giving $\frac{a}{4} \times 2\sqrt{\frac{a}{g}} = \frac{1}{2}\sqrt{ag}$	M1	Attempt to find v. Must be from SHM equation.	
		F1	Use of **their** ω	
		A1	cao	
			total	16

Q 2		mark		sub
(a) (i)	$[a] = [x] = \text{L} \quad [v] = \text{L T}^{-1}$ $\text{L}^2\,\text{T}^{-2} = [\,\omega^2\,]\,\text{L}^2$ so $[\,\omega\,] = \text{T}^{-1}$	B1 M1 E1	 Equating [Award max of 2 if any units used instead of dimensions]	3
(ii)	angular speed frequency	B1	cao. Accept ang vel. Accept examples e.g. pulse rate	1
(b) (i)	$F = ky \Rightarrow 0.015 \times 9.8 = 0.098k$ $\Rightarrow k = 1.5$	M1 E1	$F = mg$ (0.147) and $y = 0.098$ in $F = ky$. Give for $k = \dfrac{0.015g}{0.098}$ seen or implied. Fully explained. Accept $F = ky$ not explained.	2
(ii)	N2L $\downarrow$ $0.015 \times 9.8 - (0.098 + x) \times 1.5 = 0.015\ddot{x}$ $\Rightarrow \ddot{x} + 100x = 0$	 M1 B1 A1 E1	 N2L applied including attempts at weight and upthrust Upthrust term. Allow $\pm(0.098 + x) \times 1.5$. All correct including signs. Accept $x \uparrow$ and using y Clearly shown with given x.	4
(iii)	$v^2 = 100\left(0.02^2 - 0.01^2\right)$ $\Rightarrow v = 0.1732...$ so 0.173 m s^{-1} $\left(\sqrt{0.03}\right)$ $x = 0.02\cos 10t$ we need $-0.01 = 0.02\cos 10t$ $\Rightarrow \cos 10t = -0.5$ $\Rightarrow 10t = \dfrac{4\pi}{3}$ so $t = \dfrac{2\pi}{15}$ (= 0.418879… so 0.419 s (3 s. f.) 	M1 A1 B1 M1 A1	Use of this result or differentiation etc [Using a W-E approach M1 for GPE, KE and WD against F term attempted. A1] [If time found first,: M1 for $\dot{x}$ (**their** time) used. A1] Or equivalent Equating in **their** expression for x. Allow sign error. cao [If graphical method used: B1 shape; B1 x = -0.01 line; B1 cao]	5
			total	15

Q 3		mark		sub
(i)	No tangential acceleration	E1		1
(ii)	$R = mr\omega^2 = 0.3 \times 0.4 \times 100$ $= 12$ so 12 N	M1 A1	N2L radially. (Award for v^2/r with $v = 10$)	2
(iii)	$\uparrow \quad F = 0.3g$ (2.94) $F \le \mu R$ so $\mu \ge \frac{g}{40}$ so least value is $\frac{g}{40}$ (0.245)	B1 M1 A1	Accept inequality Accept =. Only vertical F considered. Use **their** R from (ii). Accept inequality inc strict. FT value of R from (ii).	3
(iv)	$R = mr\omega^2 = 0.12(10+5t)^2$ $= 3(2+t)^2$	M1 E1	N2L radially and substitute for ω . (Not v^2/r with $v = (10 + 5t)$) Clearly shown	2
(v)	$F = mr\dot{\omega} = 0.3 \times 2 = 0.6$ $F \le \mu R \Rightarrow \mu \ge \frac{0.6}{3(2+t)^2} = \frac{1}{5(2+t)^2}$ We need the greatest value μ can take which is when $t = 0$. This gives $\mu \ge 0.05$.	B1 M1 E1 E1 E1	N2L transverse direction Condone = and $\le$. R must be correct. F must be attempted from consideration of transverse motion. Clear explanation required Accept =. Dependent on correct reason given.	5
			total	13

Q 4		mark		sub
(i)	$V=\int_0^h \pi\left(r^2-x^2\right)\mathrm{d}x$	M1	Accept π omitted.	
	$=\pi\left[r^2x-\frac{1}{3}x^3\right]_0^h$	A1	At least one term correct. Limits not required. Accept any multiple	
	$=\frac{\pi h}{3}\left(3r^2-h^2\right)$	A1	Any form	
	$V\bar{x}=\int_0^h \pi x\left(r^2-x^2\right)\mathrm{d}x$	M1	Accept π omitted.	
		B1	RHS correct	
	$=\pi\left[\frac{r^2x^2}{2}-\frac{x^4}{4}\right]_0^h$	B1	RHS has at least one term correct. Limits not required. Accept any multiple	
	$=\frac{\pi h^2}{4}\left(2r^2-h^2\right)$	B1	RHS correct in any form. Accept any multiple.	
	$\bar{x}=\dfrac{\frac{\pi h^2}{4}\left(2r^2-h^2\right)}{\frac{\pi h}{3}\left(3r^2-h^2\right)}=\frac{3h}{4}\left(\frac{2r^2-h^2}{3r^2-h^2}\right)$	E1	Clearly shown	8
(ii)	Put $h=r$ and $\bar{x}=\frac{3r}{4}\times\frac{1}{2}=\frac{3r}{8}$	E1	Must see $h=r$ implicit or explicit. Accept no comment on result	1
(iii)	$V=\frac{11\pi r^3}{24}$	E1	Must see full substitution and working	
	$\bar{x}=\frac{21r}{88}$	E1	Must see full substitution and working	
			[SC 1: Both substituted, neither worked]	2
(iv)	$V_B=\frac{2\pi r^3}{3}-\frac{11\pi r^3}{24}=\frac{5\pi r^3}{24}$	B1		
	$\frac{3r}{8}\times\frac{2\pi r^3}{3}=\frac{11\pi r^3}{24}\times\frac{21r}{88}+\frac{5\pi r^3}{24}\bar{x}$	M1	Expression involving $\bar{x}$	
		A1	LHS or 1st term RHS correct	
	$\Rightarrow \frac{1}{4}r^4=\frac{7}{64}r^4+\frac{5r^3}{24}\bar{x}$ so $\bar{x}=\frac{27}{40}r$	A1	Or equivalent. cao.	
	distance is $\frac{27r}{40}-\frac{r}{2}=\frac{7r}{40}$	F1	FT subtraction of $0.5r$ or use of $\bar{x}+0.5r$ above.	
			[If fresh calculation started. B1 obtaining $V_B=5\pi r^3/24$ M1 A1 obtaining $\int \pi y^2 x\,\mathrm{d}x=9\pi r^4/64$ A1 $\bar{x}=27r/40$ cao F1 $\bar{x}=7r/40$ FT subtraction of $0.5r$ or use of $\bar{x}+0.5r$ above.]	5
			total	16

2609 - Mechanics 3

General Comments

There seemed to be a wide range in the ability of the candidates. Also, although many produced reasoned solutions to every question, others did not seem to be familiar with all of the topics and produced good answers to only one or two questions. Some candidates were unable to do much on any question.

As in earlier sessions, although there were many very well presented scripts, quite a few candidates suffered from their poor presentation, lack of diagrams and lack of indication of methods. For instance, in Q1 many candidates wrote $2h^2$ when they meant $(2h)^2$ and, although they might well claim that 'I know what I mean', quite few deceived themselves into using $2h^2$ instead of $4h^2$. Candidates should know that when asked to *show* a given result, a single step is rarely sufficient.

Unfortunately, Q1 presented rather greater problems to many of the candidates than were intended or anticipated. The accumulated effect of their errors seemed to cause some candidates to spend too long on this question, which may have made it hard for them to finish the paper. Account was taken of this when setting the grade thresholds.

There were some very pleasing solutions to every question and the general standard on the volume and centre of mass question was particularly high.

Comments on Individual Questions

1) **The tension and energy in a stretched elastic band**

Although many candidates made fundamental mistakes, some of them doing so in more than one part, quite a few candidates scored at least half marks despite their errors. There were a number of very neat complete solutions. Quite a common error was to confuse *stiffness* with *modulus of elasticity*.

(i) Most candidates scored all the marks for this section, showing that they understood the initial situation. Some used an unnecessary division of the band into parts corresponding to each side of the square.

(ii) Very many candidates failed to establish the given expression for the extension in the band because they made no reference to its unstretched length; it was not clear whether this was because they (wrongly) thought it was too obvious to mention or whether having found the given result was the length of the band below the lower pegs they thought they had finished.

There were many poor attempts at finding the vertical distance h. Many candidates thought that the equilibrium position could be found by equating the elastic potential energy gained to the gravitational potential energy lost. Many others tried to equate the weight of the particle to some vertical force but took this vertical force to be T or $2T$, where T is the tension in the band. Only a minority drew a clear diagram and realized that T had to be resolved. Many of these errors gave expressions that were obviously wrong and some candidates seemingly spent a lot of time trying to find out why.

(iii) Most candidates rightly used an energy method here. Some took the equilibrium position to be the position of the particle with $h = 0$. Relatively few candidates included all the terms in the work – energy equation. A few omitted both elastic potential energy terms but many omitted the gravitational potential energy and/or one of the elastic potential energy terms.

2) (a) **Dimensional analysis**

(i) Most candidates could write down the required dimensions but by no means all could establish the dimensions of ω and quite a lot of poor notation was seen. The most common error was to argue that the term in brackets had dimensions $L^2 - L^2$, which was dimensionless. Taking a to be acceleration was quite common.

(ii) Some candidates omitted this part. Most who answered it correctly gave frequency or angular speed. Some interesting specific examples were seen such as pulse rate.

(b) **The simple harmonic motion of a fisherman's float**

(i) Quite a few candidates could not come up with a complete argument to establish the constant of proportionality in this example of direct proportion, even with the answer given. Many found plausible combinations of the given values that came to 1.5 but could not say why they were relevant.

(ii) Many candidates knew exactly what to do and did it well. Some did not take x to be positive downwards and they mostly obtained a wrong expression for y in terms of x; these candidates, and many with x correctly defined, made mistakes with the sign of at least one term. Many candidates with a sign error slipped in the necessary 'adjusting' sign change without comment in order to obtain the given result. Quite a few candidates did not use Newton's second law properly and tried to establish the equation of motion without reference to the weight of the float.

(iii) There were many good answers to this part, especially for *v*. There were some sign errors in the working for *t* and many candidates who avoided that error still went for the time when the float was going *upwards* at the required height. Some candidates tried an energy approach for *v* but they typically forgot the work done against the force *F*.

3) **Motion in a horizontal circle**

There were some very good answers to this question and quite a few candidates (often whole centres) did well at all parts except (i) and the last part of (v). However, on the whole this was the least well answered question on the paper with many candidates (often whole centres) obviously not being familiar with a situation of this sort. It also seemed that many candidates were not familiar with the acceleration towards the centre being expressed in terms of the angular speed.

(i) There were relatively few clear correct answers to this part. Some candidates may have realized that their statements depended on constant angular speed but they did not say so. A common reason given was that there was no friction because the particle was not slipping.

(ii) Many candidates correctly found the normal reaction as the force towards the centre of the circle. One quite common error was to take the acceleration to be v^2/r with v given the value of the angular speed.

(iii) Many candidates obtained full marks for this. The most common error was to take the frictional force to be the force towards the centre and the normal reaction to be the weight instead of the other way round.

(iv) Many candidates could see what to do but not many of them could argue efficiently that $0.3 \times 0.4 \times (10 + 5t)^2 = 3(2+ t)^2$, with most electing to expand the bracket first and then factorise later.

(v) Only a minority of candidates realized that the frictional force could be found by applying Newton’ s second law to the transverse motion and a common error was to assume that the frictional force was the weight of the particle. Relatively few candidates could argue the last part properly. Those who saw that μ takes its least value when $t = 0$ and argued from there often scored both marks; those who spotted that the given result could be obtained by putting $t = 0$ often did this without much or any explanation and rarely scored marks.

4) **A volume of revolution and centre of mass obtained using calculus**

There were many complete or almost complete solutions to this question. Relatively few candidates thought they were dealing with areas and the general standard of the working was high. A few candidates (often whole centres) had obviously not prepared this topic and scored very few marks, making elementary mistakes such as integrating the constants as if they were variables. The following comments apply to the majority of the candidates who understood essentially what should be done.

(i) There were a few slips with the constants and the limits of integration and the arithmetic. Many candidates lost the final *show* mark because they did not properly establish how their expression produced the given result.

(ii) Many candidates stated the given result but did not say that this must correspond to $h = r$ or did not show that it worked.

(iii) Again, most candidates knew what to do and did it but others did not do enough to establish the given result.

(iv) Some candidates omitted this part. Surprisingly, many candidates elected to substitute new limits into their expressions in (i) instead of using the results of parts (ii) and (iii). Most who tried to use parts (ii) and (iii) produced a correct expression as long as they had the correct volume for solid B. A very common mistake was to leave the final answer as a distance from O instead of subtracting $\frac{1}{2} r$ to find the required distance from the plane face of B.